Design Theory Tirgul

Function Dependencies

- Let R be a schema of a relation, that contains the sets of attributes X and Y. Let r be an *instance* of R.
- **Definition:** **X→ Y** holds in r if **for every** two tuples s and t in r,
  - if s[X]=t[X] then s[Y]=t[Y]
- **Alternative (Equivalent) Definition:** **X→ Y** holds in r if **there do not exist** tuples s and t in r,
  - Such that s[X]=t[X] and s[Y]≠t[Y]

Is it possible?

- Let R be a schema of a relation containing all attributes in X and Y. Let r be an instance of R.
- In the following case, determine whether:

1. $X \rightarrow Y$ certainly holds in r
2. $X \rightarrow Y$ certainly does not hold in r
3. $X \rightarrow Y$ may or may not hold in r

- r contains only 1 tuple

Is it possible?

- Let R be a schema of a relation containing all attributes in X and Y. Let r be an instance of R.
- In the following case, determine whether:
  1. $X \rightarrow Y$ certainly holds in r
  2. $X \rightarrow Y$ certainly does not hold in r
  3. $X \rightarrow Y$ may or may not hold in r

- No two tuples in r are equal on X

Is it possible?

- Let R be a schema of a relation containing all attributes in X and Y. Let r be an instance of R.
- In the following case, determine whether:
  1. $X \rightarrow Y$ certainly holds in r
  2. $X \rightarrow Y$ certainly does not hold in r
  3. $X \rightarrow Y$ may or may not hold in r

- all tuples in r are equal on X, and r contains more than one tuple

Is it possible?

- Let R be a schema of a relation containing all attributes in X and Y. Let r be an instance of R.
- In the following case, determine whether:
  1. $X \rightarrow Y$ certainly holds in r
  2. $X \rightarrow Y$ certainly does not hold in r
  3. $X \rightarrow Y$ may or may not hold in r

- all tuples in r are equal on X, r contains more than one tuple and all attributes in R appear in X or Y

Is it possible?

- Let R be a schema of a relation containing all attributes in X and Y. Let r be an instance of R.
- In the following case, determine whether:
  1. $X \rightarrow Y$ certainly holds in r
  2. $X \rightarrow Y$ certainly does not hold in r
  3. $X \rightarrow Y$ may or may not hold in r

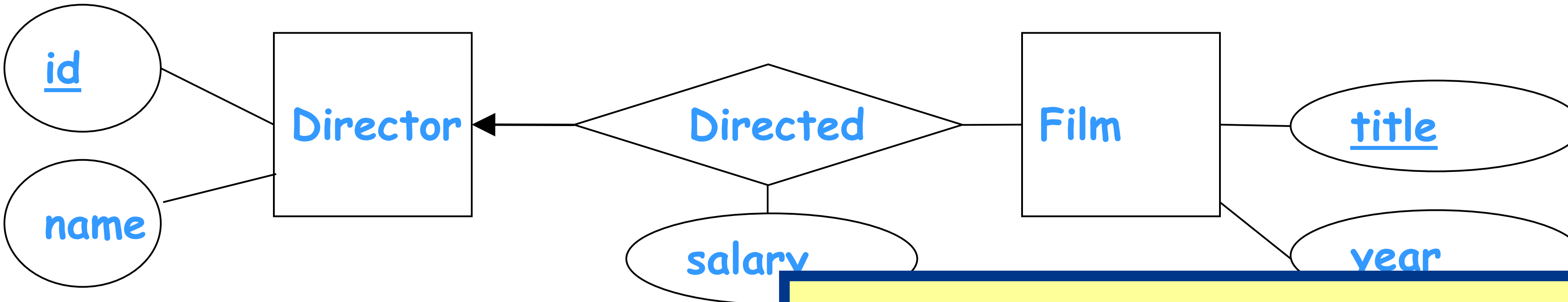
- all tuples in r are equal on Y

Is it possible?

- Let R be a schema of a relation containing all attributes in X and Y. Let r be an instance of R.
- In the following case, determine whether:
  1. $X \rightarrow Y$ certainly holds in r
  2. $X \rightarrow Y$ certainly does not hold in r
  3. $X \rightarrow Y$ may or may not hold in r

- r contains more than 1 tuple, and all tuples in r differ on Y

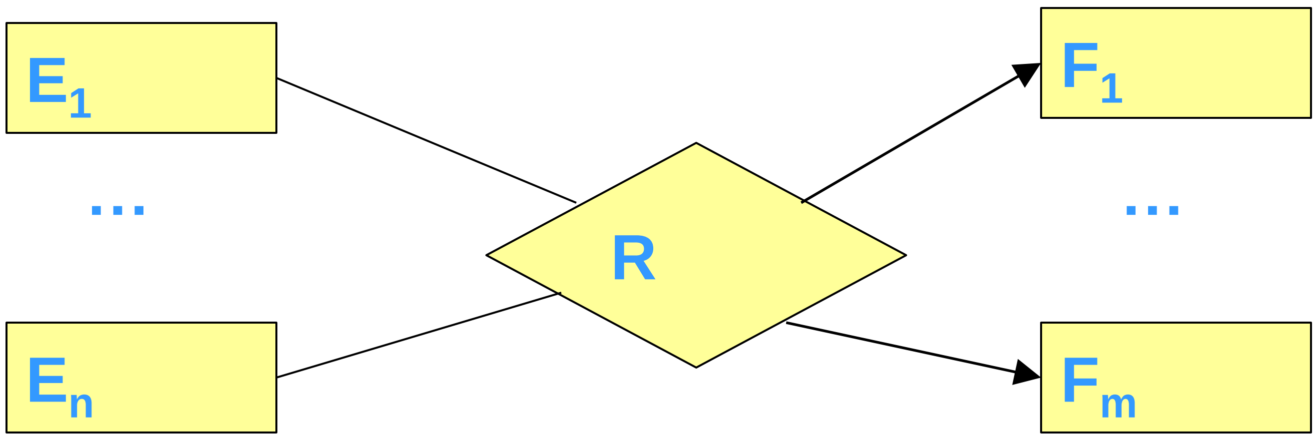
Intuition 1: Keys



- Director(id, name)
- Film(title,year)
- Directed(title,salary,id)

- In Director, the FD id→name must hold
- What FDs must hold in Film? In Directed?
- **title → year**
- **title → salary, id**

Intuition 2: Multiway Relations



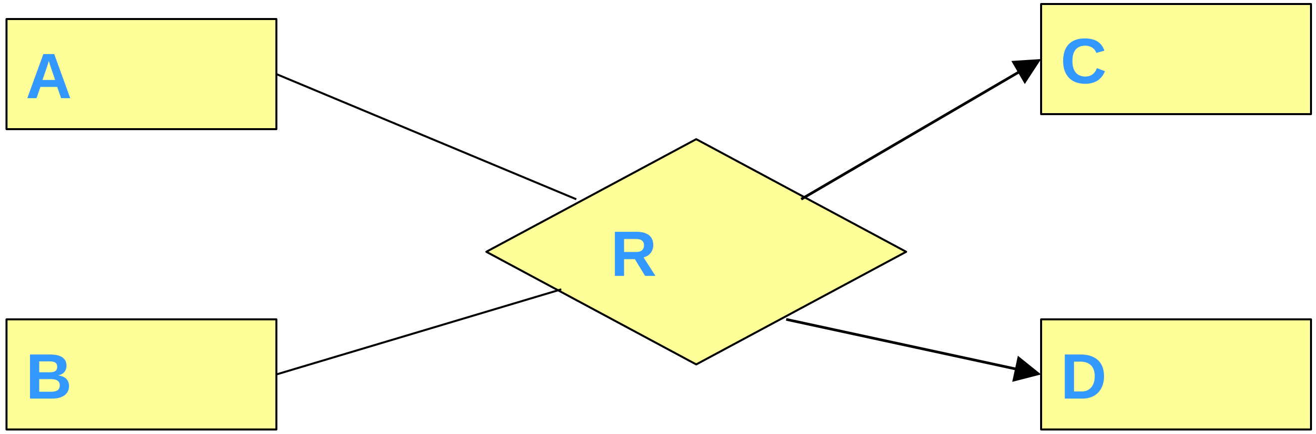
For any $1<=i<=m$, for any tuple of entities $e_1,\ldots,e_n,f_1,\ldots f_{i-1},f_{i+1},\ldots,f_m$ there is at most one f_i, such that $e_1,\ldots,e_n,f_1,\ldots,f_m$ are connected by R

R will be translated into a table $R(E_1,\ldots,E_n,F_1,\ldots,F_m)$

The following FDs should hold over R: For all i

$E_1,\ldots,E_n,F_1,\ldots,F_{i-1},F_{i+1},\ldots,F_m \rightarrow F_i$

Example



In the Relation R(A,B,C,D), which functional dependencies should hold?

ABC → D

ABD → C

FD in SQL

- Let R(A,B,C,D,E) be a table
- Write an SQL query that returns an empty relation if and only if AB→CD holds in R.
- **SELECT ***

  FROM R R1, R R2

  WHERE R1.A = R2.A and R1.B = R2.B and

  (R1.C<>R2.C or R1.D<>R2.D)

FD in SQL

- How would you enforce this constraint in the database?

- **As a "FOR EACH STATEMENT", "AFTER" trigger that runs the query from the slide before and throws an exception if the result is not empty**

Prove these

- Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
- Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
- Pseudo Transitivity: If $X \rightarrow Y$ and $YW \rightarrow Z$, then $XW \rightarrow Z$

- **Exercise:** Derive each of these rules from Armstrong's axioms

Prove these

- Union: If X$\rightarrow$ Y and X$\rightarrow$ Z, then X$\rightarrow$ YZ

1. **X$\rightarrow$ Y (given)**
2. **X$\rightarrow$ XY (augmentation of 1)**
3. **X$\rightarrow$ Z (given)**
4. **XY$\rightarrow$ ZY (augmentation of 3)**
5. **X$\rightarrow$ YZ (transitivity 2, 4)**

Prove these

- Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

1. **$X \rightarrow YZ$ (given)**
2. **$YZ \rightarrow Y$ (reflexivity)**
3. **$X \rightarrow Y$ (transitivity 1, 2)**

Prove these

- Pseudo Transitivity: If X$\rightarrow$ Y and YW$\rightarrow$ Z, then XW $\rightarrow$ Z

1. **X$\rightarrow$ Y (given)**
2. **XW$\rightarrow$ YW (augmentation of 1)**
3. **YW$\rightarrow$ Z (given)**
4. **XW$\rightarrow$ Z (transitivity of 2, 3)**

The Closure of a set of Attributes

- The closure of the attributes X, with respect to the FDs F is denoted $X^+_F = \{A \mid F \vdash X \rightarrow A\}$
  - Note: $\vdash$ means *provable using Armstrong's Axioms*
  - If F is clear from the context, we simply write X^+
- Lemma: Let Y be a set of attributes. Then, $Y \subseteq X^+$ if and only if $F \vdash X \rightarrow Y$
- Proof in notes of lecture slide 45

Axiom of Difference

- Axiom of difference:
  - If $XW \rightarrow YW$ and $W \rightarrow Y$ then $X \rightarrow W$
- Prove this axiom using Armstrongs Axioms or show that it is not sound
- **Not sound. Example:**

X	Y	W
a	b	c1
a	b	c2