Graphs & Trees

Intro2CS – week 8-9

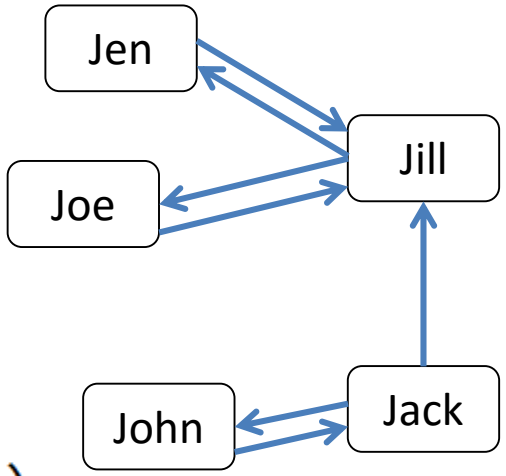
General Graphs

- Nodes (Objects) can have more than one reference.
- Example: think of implementing a Class “Person”
- Each Person has a list of friends, which are also of the same “Person” class.
- A “Social Network”

```
class Person:
    def __init__(self,name):
        self.__name = name
        self.__list_of_friends=[]

    def add_friend(self,other):
        self.__list_of_friends.append(other)

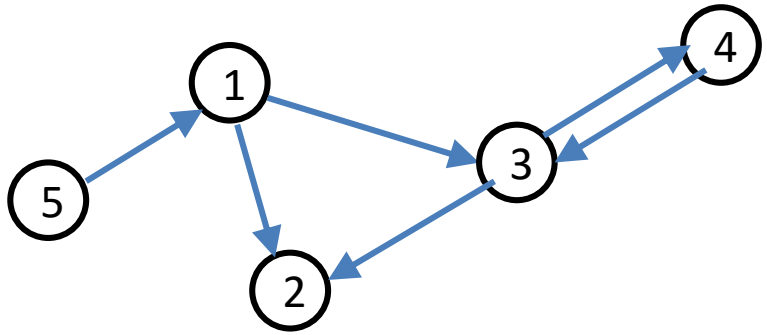
    ...
```



Graphs

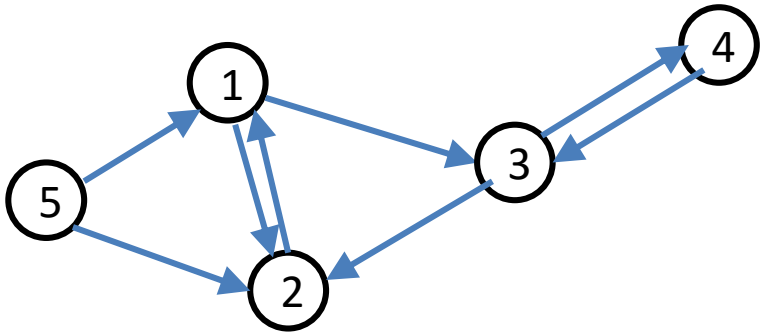
- We generally think of the structure of these as a general (directed) Graph G=(V,E)
- V is called the set of Nodes or Vertices
- $E \subseteq V \times V$ is called the set of Links, or Edges

$(v_1, v_2) \in E$ If and only if object $v_1 \in V$ has a link to $v_2 \in V$



Paths

- A Path in the graph (of length n) is a sequence of vertices $P = (v_1, v_2, \dots, v_n)$ such that
- $\forall i \in \{1,2 \dots, n\} \quad v_i \in V$
- $\forall i \in \{1,2 \dots, \mathrm{n}-1\} \ (v_i, v_{i+1}) \in E$



Traversing general graphs

- Suppose we wish to visit all nodes in a graph and print their data (only once!)

- How do we do this?

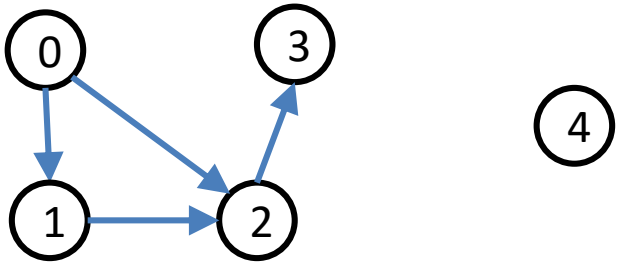
```
class Node:
    def __init__(self,data=None):
        self.data = data
        self.neighbors = set()

    def add_edge(self,other):
        self.neighbors.add(other)
```

First attempt

```
def visit_all_from(node):

    print(node.data)
    for neighbor in node.neighbors:
        visit_all_from(neighbor)
```




```
nodes = [Node(i) for i in range (5)]

nodes[0].add_edge(nodes[1])
nodes[0].add_edge(nodes[2])
nodes[1].add_edge(nodes[2])
nodes[2].add_edge(nodes[3])

print("*** From Node 0:")
visit_all_from(nodes[0])
print("\n*** From Node 4:")
visit_all_from(nodes[4])
```

```
*** From Node 0:
0
1
2
3
2
3

*** From Node 4:
4
```

What went wrong?

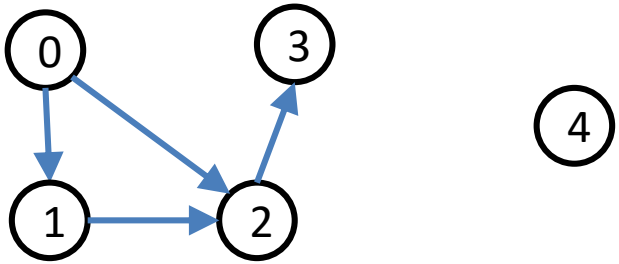
What if we had a loop in the graph?

Fixing the problem

```
def visit_all_from(node):
    visited = set()
    _visit_helper(node,visited)


def _visit_helper(node,visited):
    if node in visited:
        return

    visited.add(node)
    print(node.data)
    for neighbor in node.neighbors:
        _visit_helper(neighbor,visited)
```

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*** From Node 0:
0
2
3
1

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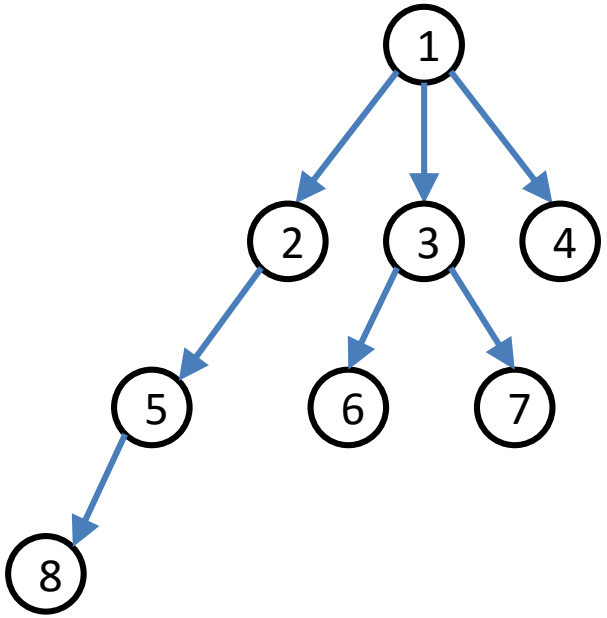
4
```

Trees

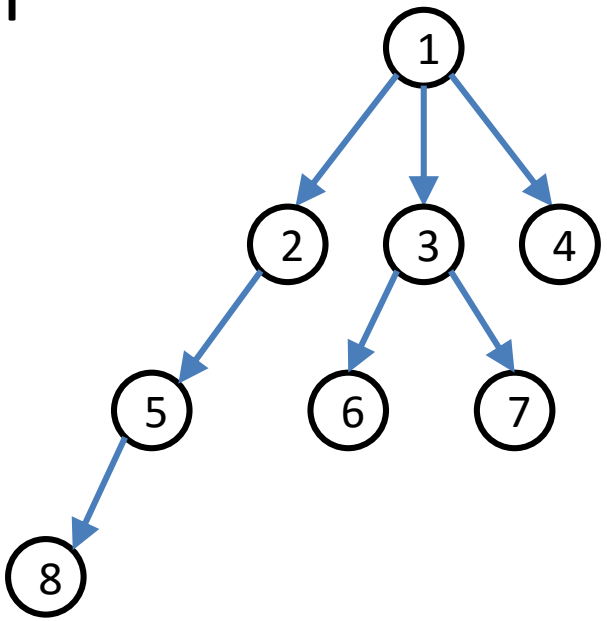
- Directed Rooted Trees are one particularly useful class of Graphs.



- They have a special node called “the root”
- There is exactly one path from the root to every other node in the tree.
  - No Cycles!

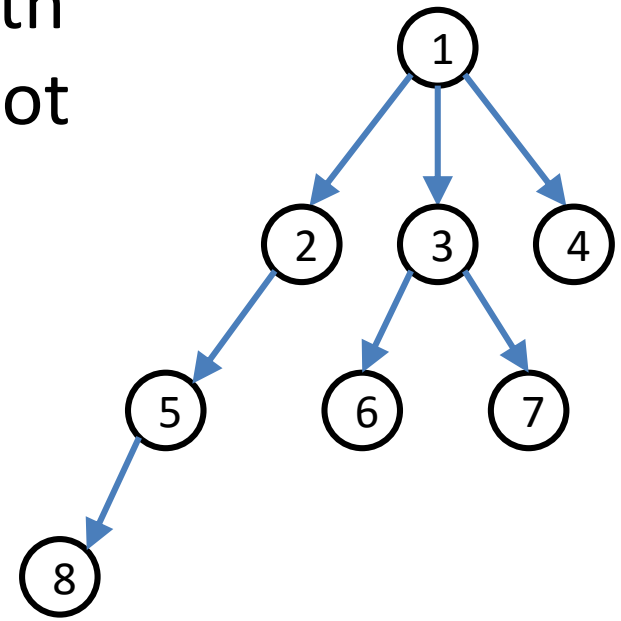
Tree Terminology

- A node that is directly linked from v is often called a child of v (and v is called the parent)
  - Example: 6,7 are children of 3
- A Node that has no outgoing links is called a Leaf.
  - Example: 4,6,7,8 are all leaves



Tree Terminology

- The height of a node is the length of its path from the root (the root is of height 0).
- Example: Node 5 has height 2.

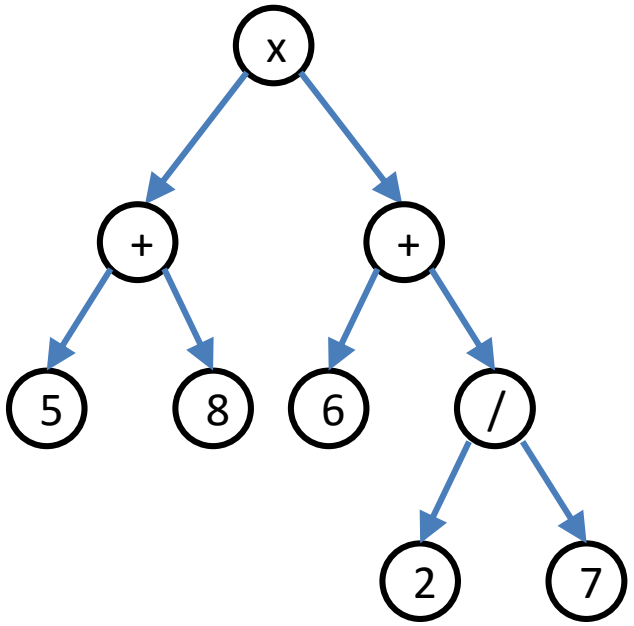


- The height of the tree is the length of the longest path.
- Example: the tree here has height 3 (due to the path to node 8)

Example: Trees that represent expressions

- One thing to naturally represent with trees is mathematical expressions:

(5 + 8) x (6 + (2 / 7))



- Leaves are all literals, internal nodes are operators
- Notice that the order of the children matters

In Python

```
class TreeNode:
    def __init__(self,data,left=None,right=None):
        self.data = data
        self.left = left
        self.right = right
```

```
expr1 = TreeNode("+",
                 TreeNode("*", TreeNode("3"),TreeNode("4")),
                 TreeNode("7"))
```

Computing the value of an expression

```
def compute(root):
    if root.data == "+":
        return compute(root.left)+compute(root.right)
    elif root.data == "*":
        return compute(root.left)*compute(root.right)
    elif root.data == "/":
        return compute(root.left)/compute(root.right)
    elif root.data == "-":
        return compute(root.left)-compute(root.right)
    else:
        return float(root.data)
```

```
expr1 = TreeNode("+",
                 TreeNode("*", TreeNode("3"),TreeNode("4")),
                 TreeNode("7"))
print(compute(expr1))
```

Printing the expression

```
def tree_to_expr(tree_root):
    if tree_root.left:
        return "(" + tree_to_expr(tree_root.left) + \
            tree_root.data +\
            tree_to_expr(tree_root.right) + ")"
    else:
        return tree_root.data
```

```
expr1 = TreeNode("+",
                 TreeNode("*", TreeNode("3"),TreeNode("4")),
                 TreeNode("7"))
print(tree_to_expr(expr1))
```

Polish Notation

- Polish notation (also called prefix notation) is just a different way to write mathematical expressions

- The operator is always written to the left.
- Instead of (2+3) write: (+ 2 3)

- We never need parentheses when writing this way. Order of operations is always well defined:

+ x 3 4 - 2 5

(+ (x 3 4) (- 2 5))

In “regular” notation: (3 x 4) + (2-5)

```
def tree_to_polish(tree_root):
    result = tree_root.data
    if tree_root.left:
        result += " " + tree_to_polish(tree_root.left) + \
                  " " + tree_to_polish(tree_root.right)
    return result
```

```
expr1 = TreeNode("+",
                 TreeNode("*", TreeNode("3"),TreeNode("4")),
                 TreeNode("7"))
print(tree_to_expr(expr1))
print(tree_to_polish(expr1))
```

Traversing a tree

The order of visiting a tree can be defined (recursively)

- Pre order: print the root, then print the subtrees
  - Example: when we were printing an expr in polish notation
- In order: left subtree, root, right subtree
  - Example: when we were printing a “regular expression”
- Post order: print the subtrees, then the root
  - Example: reverse polish notation

(Extra) Parsing Polish Notation

```
def _p_polish_helper(polish_expr):
    if polish_expr[0] in ("+","-","*","/"):
        arg1, remainder = _p_polish_helper(polish_expr[1:])
        arg2, remainder = _p_polish_helper(remainder)
        return TreeNode(polish_expr[0], arg1, arg2), remainder
    else:
        return TreeNode(polish_expr[0]), polish_expr[1:]


def parse_polish(text):
    tree, leftovers = _p_polish_helper(text.split(" "))
    if leftovers:
        print("There were extra symbols left over.")
    return tree
```

Trees – Twenty Questions

Does it have four legs? y

Is it really large? y

Is it an elephant? n

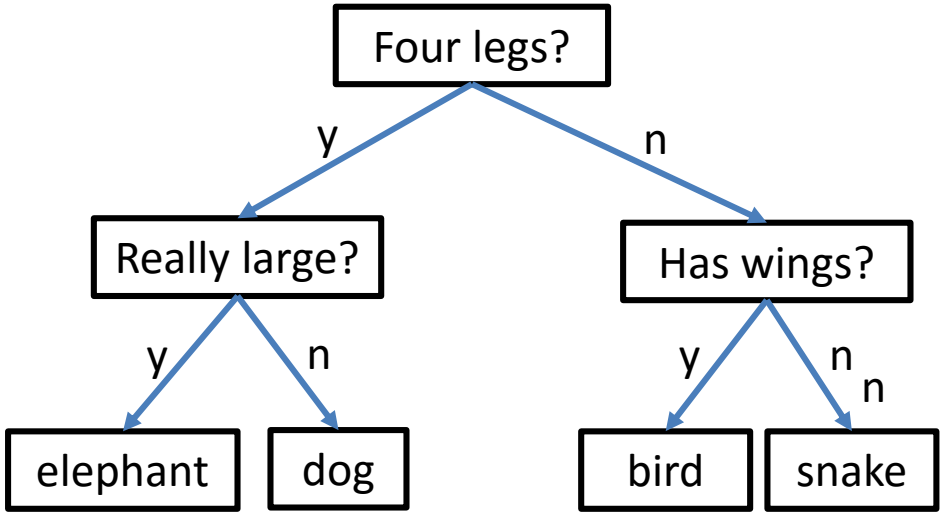
I guessed wrong.

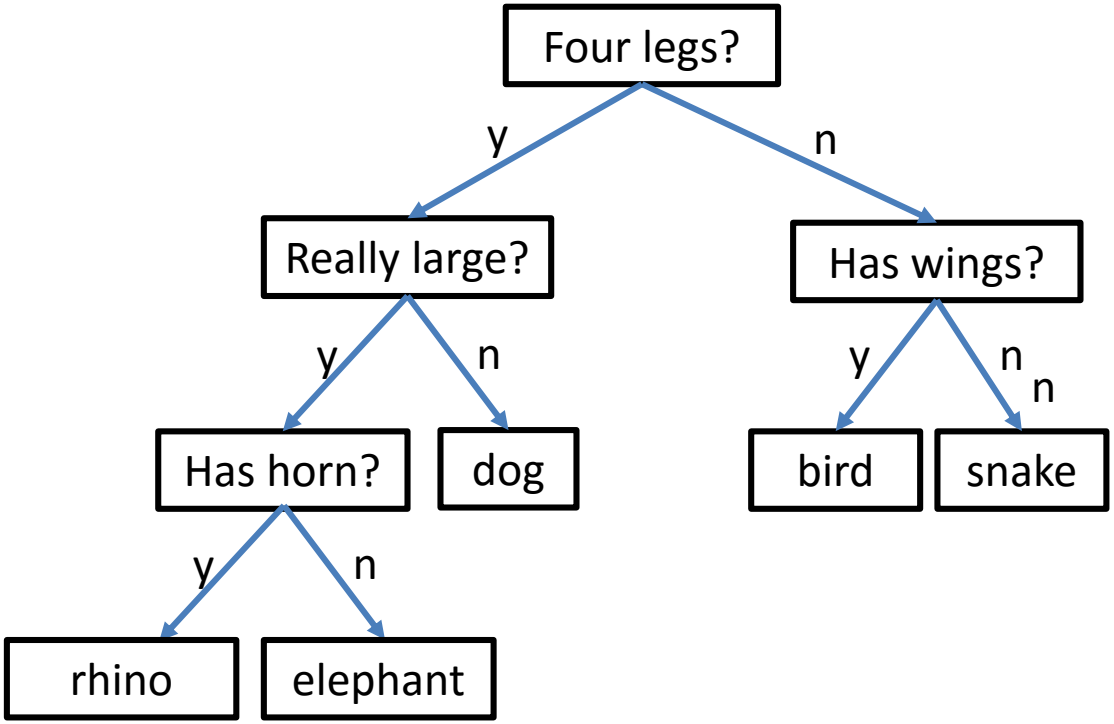
What did you have in mind? a rhino

Please enter a question to differentiate between an elephant and a rhino: does it have a horn?

an elephant. does it have a horn? n

- Do you want to play again? y


Four legs?
y
n
Really large?
Has wings?
y
n
y
n
n
elephant
dog
bird
snake


Four legs?
y
n
Really large?
Has wings?
y
n
y
n
n
Has horn?
dog
bird
snake
y
n
rhino
elephant

```
YES = 'y'
NO = 'n'


def get_yes_no_answer(question):
    while True:
        answer = input(question + " ")
        if answer == YES:
            return True
        elif answer == NO:
            return False
        else:
            print ("I did not understand.")
```

```
class Question:
    def __init__(self,question_text, yes_answer=None, no_answer=None):
        self.__question_text = question_text
        self.__on_yes_answer = yes_answer
        self.__on_no_answer = no_answer
```

```
def ask_question(self):
    if self.__on_yes_answer is not None:
        if get_yes_no_answer(self.__question_text):
            self.__on_yes_answer.ask_question()
        else:
            self.__on_no_answer.ask_question()
    else:
        if get_yes_no_answer("is it " + self.__question_text + "?"):
            print("I knew it!")
        else:
            print("I guessed wrong.")
            self.__add_new_question()
```

```
def __add_new_question(self):
    other_answer = input("What did you have in mind? ")
    new_question = input("Please enter a question to differentiate between " +\
                         self.__question_text + " and " + other_answer + ": ")
    if get_yes_no_answer(self.__question_text+". "+new_question):
        self.__on_yes_answer = Question(self.__question_text)
        self.__on_no_answer = Question(other_answer)
    else:
        self.__on_yes_answer = Question(other_answer)
        self.__on_no_answer = Question(self.__question_text)
    self.__question_text = new_question
```

```
def play_twenty_questions():
    root_question = Question("a sparrow")

    print("Let's play twenty questions. Think of something...")
    print("I'll guess it!")
    root_question.ask_question()
    while get_yes_no_answer("\n\nDo you want to play again?"):
        root_question.ask_question()

    print("\n\nHere are all the possible answers entered into the game:")
    root_question.print_all_answers()


if __name__ == "__main__":
    play_twenty_questions()
```

```
def print_all_answers(self):
    if self.__on_yes_answer is None:
        print(self.__question_text)
    else:
        self.__on_no_answer.print_all_answers()
        self.__on_yes_answer.print_all_answers()
```