Using Exact Sciences Modeling Tools to Understand Social Phenomena

Course #: 55772

The Simplex Representation

Recommended background reading - the Wikipedia page is short but very accurate and rich: https://en.wikipedia.org/wiki/Ternary_plot

Simplex representation in the context of Epstein Model:

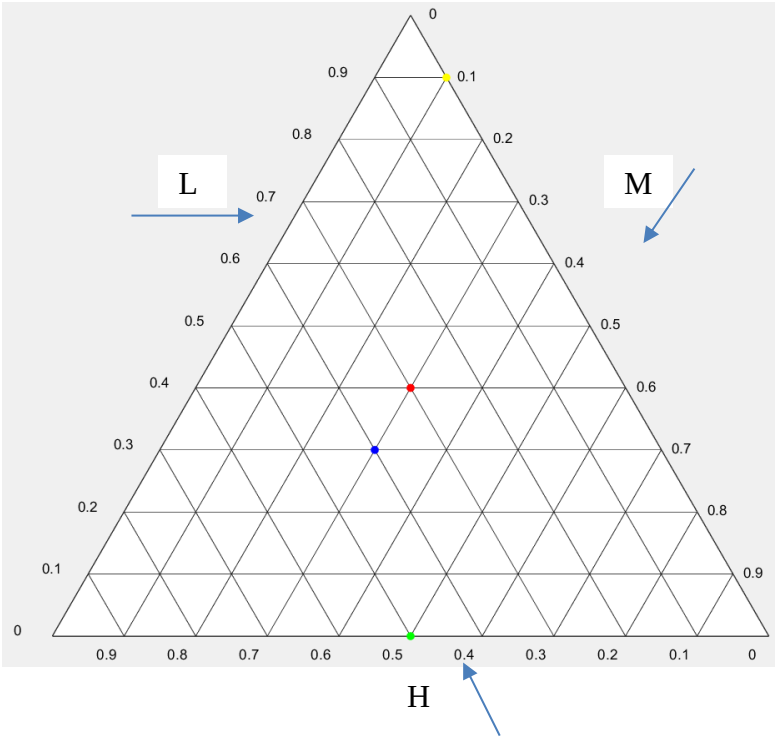
As an example, let's look on the memory of an individual, Eve, and assume that her memory goes 10 time steps back. i.e., Eve remembers 10 games she played.
Examples of Eve's possible memory:

- LLLHHHMMMM
- LLLLLLLLLM
- HLMHLMHLMH
- MMMMMHHHHH

Since our population are rational – i.e. their decisions are based solely on their memory – Eve decision is done based on this memory. Hence, we can decode the whole population by their memories, and this is what the simplex does.

Let's put Eve and three of her friends on the simplex by translating their memories to a vector with 3 entries (H,M,L):

- LLLHHHMMMM ➔ 3H,4M,3L ➔ (0.3,0.3,0.4)
- LLLLLLLLLM ➔ 0H,1M,9L ➔ (0,0.1,0.9)
- HLMHLMHLMH ➔ 4H,3M,3L ➔ (0.4,0.3,0.3)
- MMMMMHHHHH ➔ 5H,5M,0L ➔ (0.5,0.5,0)



Note that in our model, the memory has no order: Memory of LM is equivalent to ML.

Make sure you understand:
1. How each axis is related to each grid lines.
2. How each point represents one of the four memories above.

The colored areas in Epstein model:

Recall from class - How an individual makes the decision based on her memory? Make a demand that maximizes your expected payoff given your expectations about the opponent's behavior (based on the last m matches). If several demands maximize the expected payoff, they are chosen with equal probability.

Example: If Eve faced the following history:
HHHMHHLLLL ➔ 5H, 1M, 4L
If she chooses L, she will be paid: 0.5*30+0.1*30+0.4*30 = 30
If she chooses M, she will be paid: 0.5*0+0.1*50+0.4*50= 25
If she chooses H, she will be paid: 0.5*0+0.1*0+0.4*70= 28

Hence, the rational decision in our model is to choose L!

Since the rational decision is based only on the memory, and each point in the simplex a memory – each point in the simplex have one rational decision. The example above is a point on the simplex, and every individual that will have such a memory will always choose L. So this point on the simplex is always colored L.

To conclude: point on the simplex is a memory state have one rational decision (H/L/M) which we give a color.

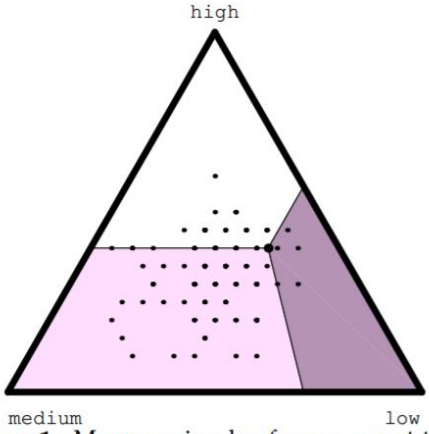


Figure 1: Memory simplex for one agent type

The black dots are individuals' memory (some might overlap)
The colored areas are the rational decision must be made by individuals on those areas, i.e., the best choice according to Eve's memory.
Note that ,as you can see in the plot, the colors are NOT symmetric, which is inherent in our model: For instance, (0.3,0.3,**0.4**) Is not symmetric with (**0.4**,0.3,0.3) and (0.3,**0.4**,0.3):

	3H,3M,4L	**4H,3M,3L**	**3H,4M,3L**
H	3*0+3*0+4*70 = 280	4*0+3*0+3*70 = 210	3*0+4*0+3*70 = 210
M	3*0+3*50+4*50 = **350**	4*0+3*50+3*50 = **300**	3*0+4*50+3*50 = **350**
L	3*30+3*30+4*30 = 300	4*30+3*30+3*30 = **300**	3*30+4*30+3*30 = 300
	Best choice M	Best choice M or L	Best choice M

So, there is no symmetry in the best decision in the simplex! And therefore the colored areas are not symmetric.

Implementation:

There are many different ways to implement the simplex plot.
From my experience, working on graphical output can consume a lot of time, and since this is a common plot in different areas (chemistry, social science etc.) I would recommend for you to use some open-source packages.

Here are some examples that I used on Matlab (they have a nice demo example for you to try):

- https://www.mathworks.com/matlabcentral/fileexchange/7210-ternary-plots
- https://www.mathworks.com/matlabcentral/fileexchange/2299-alchemyst-ternplot

And on python (check dependencies):

- https://plot.ly/python/ternary-plots/
- https://github.com/marcharper/python-ternary

If you work on other programming languages you can look for other open-source libraries under the names: ternary plot, ternary graph, triangle plot, simplex plot, de Finetti diagram

Two notes about the homework:

1. There is no need to add the colored area to your plots.
2. Since working on graphical output might take a lot of time, for the simplex representation, you are encouraged hereby to share on the moodle forum - code you found useful or a code that you wrote for the use of other students.

Good Luck!