CS 67800, Spring 2017/18

Problem Set 2: Markov Networks

Submission Date : Thursday 17/5/18, 23:59

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1. Prove that for a Bayesian Network $\mathcal{G}$: $\mathcal{I}(\mathcal{M}[\mathcal{G}]) \subseteq \mathcal{I}(\mathcal{G})$, where $\mathcal{M}[\mathcal{G}]$ is the moral graph of $\mathcal{G}$. In other words: $\mathcal{M}[\mathcal{G}]$ is an I-map for $\mathcal{G}$.
   Bonus: Prove that $\mathcal{M}[\mathcal{G}]$ is a minimal I-map.

2. (a) Show an example for a distribution P (not necessarily positive) and a Markov Network $\mathcal{H}$, such that P satisfies $\mathcal{I}_{LM}(\mathcal{H})$, but not $\mathcal{I}(\mathcal{H})$ (i.e. not $\mathcal{I}_{sep}(\mathcal{H})$).
   (b) Show an example for a distribution P (not necessarily positive) for which the intersection property:
   $$(X \perp Y \mid Z, W) \wedge (X \perp W \mid Z, Y) \Longrightarrow (X \perp Y, W \mid Z)$$
   does <u>not</u> hold.

3. We say that a node X in $\mathcal{H}$ is *eliminable* if all its neighbors are connected (i.e. the node and its neighbors form a clique).
   (a) Show that a chordal graph has an eliminable node.
   (b) Show that if you eliminate that node (and the edges connected to it), the remaining graph is still chordal, and still has an eliminable node.

4. *(I-Maps and P-Maps)* Consider a probability distribution P over the variables A, B, C and D that satisfies only the following independencies:
   1. $A \perp C | B$.
   2. $A \perp C | B, D$.
   3. $A \perp D | B$.
   4. $A \perp D | B, C$.
   5. $B \perp D$.
   6. $B \perp D | A$.
   7. $A \perp D$.

   (a) Draw a Bayesian network that is a perfect map for P.
   (b) Does this perfect map have any I-equivalent graphs? If so, draw them. If not, explain why not.
   (c) Draw a Markov network that is a minimal I-map for P and explain why the Markov network is or is not also a perfect map.