Exercise 3

1. Use elementary row operation on A and I (that is, use the augmented matrix $[A|I]$ to find the inverse matrices of:

   1.1. $\begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$

   1.2. $\begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 3 & 1 \\ 1 & 0 & -1 & 2 \end{pmatrix}$

In Q's 2-5: solve in matrix form:

3. In exercise 1, question 4, you have shown that $\bar{v} = \begin{pmatrix} 2 \\ 7 \\ 8 \end{pmatrix}$ cannot be written as a linear combination of $\bar{u}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \bar{u}_2 = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}, \bar{u}_3 = \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}$.

   3.1. Formulate this question as a matrix equation.

   3.2. What is the determinant of the matrix whose columns are $\bar{u}_1, \bar{u}_2, \bar{u}_3$? Check your answer by direct calculation.

4. For which values of the parameter a does each of the following system has:

   4.1. A unique solution

   4.2. Infinite solutions (write down the general solution)

   4.3. No solution

Remember: If you divide by a check separately for the case where $a = 0$.

$$x + 2y + az = -3 - a$$
$$x + (2 - a)y - z = 1 - a$$
$$ax + ay + z = 6$$

5. Systems with more variables than equations ($m < n$).

5.1. Solve the following systems of equations. If there exists more than one solution, **find a basis** for the solutions set of the corresponding homogeneous system:

$$x_1 - 3x_2 + 2x_3 - x_4 + 2x_5 = 2$$
$$3x_1 - 9x_2 + 7x_3 - x_4 + 3x_5 = 7$$
$$2x_1 - 6x_2 + 7x_3 + 4x_4 - 5x_5 = 7$$

5.2. Solve the following systems of equations. If there exists more than one solution, **find a basis** for the solutions set of the corresponding homogeneous system:

$$x_1 + 2x_2 - 3x_3 + 4x_4 = 2$$
$$2x_1 + 5x_2 - 2x_3 + x_4 = 1$$
$$5x_1 + 12x_2 - 7x_3 + 6x_4 = 3$$

6. True or false?

If a system of linear equations has more variables than equations then it has infinite solutions.

7. Solve the following systems of equations. If there exists more than one solution, write the solution in parametric form:

$$x + 2y - 3z = 1$$
$$2x + 5y - 8z = 4$$
$$3x + 8y - 13z = 7$$

8. Solve the following system of equations. If there exists more than one solution, write the solution in parametric form:

$$x + 2y - 3z = -1$$
$$-3x + y - 2 = -7$$
$$5x + 3y - 4z = 2$$

9. Consider the system:

$$x + ay = 4$$
$$ax + 9y = b$$

9.1. For which value of a does the system have a unique solution?

9.2. For which pairs of values (a, b) does the system have more than one solution? How many solutions does it have then?

10. Consider the following three vectors:

$$\bar{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \bar{v}_2 = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \bar{v}_3 = \begin{pmatrix} 5 \\ -1 \\ -4 \end{pmatrix}$$

10.1. Show that they are pairwise orthogonal.

10.2. Show that they any vector $\bar{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ in $\mathbb{R}^3$ can be expressed by a linear combination of $\bar{v}_1$, $\bar{v}_2$ and $\bar{v}_3$. In other words (see chapter 4): show that they form a basis of $\mathbb{R}^3$.

10.3. Let $\bar{u} = \begin{pmatrix} 4 \\ 14 \\ -9 \end{pmatrix}$. Write down $\bar{u}$ as a linear combination of $\bar{v}_1, \bar{v}_2, \bar{v}_3$. Use the fact that the given basis is an orthogonal basis.