<u>Exercise 5-6</u>

**1.** Let $T$ be the projection transform $T: \mathbb{R}^3 \to \mathbb{R}^3$, such that $T\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) = \begin{bmatrix} v_1 \\ v_2 \\ 0 \end{bmatrix}$.

In class we showed that $T(\bar{v} + \bar{u}) = T(\bar{v}) + T\,\bar{u}$.
Complete the proof that to show that $T$ is a linear transformation.

**2.** Let $T$ be the transform $T: \mathbb{R}^3 \to \mathbb{R}^3$, such that $T\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) = \begin{bmatrix} v_1 \\ 0 \\ v_3 \end{bmatrix}$.

2.1. Describe $Im(T)$ geometrically, in words.
2.2. What is the rank of $T$?
2.3. Describe $ker(T)$ geometrically, in words.
2.4. What is $\dim\big(ker(T)\big)$?
2.5. Let $A$ be the matrix that represents $T$ in the standard basis. Write down $A$.

**3.** Decide for each of the following transformation if it is linear or not:
3.1. $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (xy, x)$
3.2. $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + y, x)$
3.3. $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\left(\binom{v_1}{v_2}\right) = \binom{v_1}{v_2} + \binom{1}{3}$
3.4. $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (|x|, y + z)$

**4.** In Ex03, Q5, you studied two systems of equations. For each system draw a schematic representation of the four fundamental subspaces:
- Clearly mark the dimensions of each subspace.
- Mark the vector $\bar{b}$ in the relevant position.

**5.** Let $A$ be a 7x9 matrix with rank 5. What are the dimensions of the four subspaces?

**6.** Let $A$ be a $m \times n$ matrix with rank $r$. Suppose there exists a vector $\bar{b}$ such that $A\bar{x} = \bar{b}$ has no solution.
6.1. What are the inequalities ($<, \leq,\ or =$) that must be true between m, n, and r?
6.2. How do you know that $A^T\bar{y} = 0$ has solutions other than $\bar{y} = 0$?