ALGEBRA EXEMPTION TEST
22/10/2013

Answer the following two questions. Each question weights 52 points (17 for each section).

(1) Let $A = \begin{bmatrix} 4 & 20 & -12 \\ 7 & 8 & 6 \\ 0 & 0 & ? \end{bmatrix}$.

(a) Find all possible values of the parameter t, such that the vector $v = \begin{pmatrix} t \\ t+3/2 \\ t+1 \end{pmatrix}$ would be an an eigenvector of A that belongs to the eigenvalue $6t$.

(b) Determine the missing element in the third row and third column of the matrix A, given that all possible values of $6t$ are indeed eigenvalues of A. You cannot use in your arguments the data given in the following section.

(c) Verify that the vector $u = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ is an eigenvector of A and determine its corresponding eigenvalue. (again: you cannot use this piece of data when explaining section b.) Determine whether A is diagonalizable or not. If it is - write down a diagonal matrix that is similar to A (a gold hint that will save you a lot of time: You can use the results of the previous sections and algebraic arguments rather than calculating the characteristic polymomial).

(2) Let V be the subspace of $M_{2\times 2}(\mathbb{R})$ (real 2×2 matrices) containing all matrices which element in the first row and first column is zero.

(a) Determine $\dim(V)$ and find a basis B of V.

(b) Let $T : \mathbb{R}^3 \to M_{2\times 2}(\mathbb{R})$ be the linear transformation defined by:

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 0 & (m+1)x + 2y + (m-1)z \\ m^2y - z & (1-m^2)y \end{bmatrix}$$

Find the representation matrix $[T]_E^E$ of the transformation T relative to the standard bases of $\mathbb{R}^3$ and $M_{2\times 2}(\mathbb{R})$. Find all values of m for which the image of T, $\mathrm{Im}(T)$, is equal to V from the previous section.

(c) For each value of the parameter m, find $\dim(\ker(T))$.

GOOD LUCK!!