Linear Algebra for Neuroscience (76992)

**Exercise 01 – Vectors, linear combinations, linear dependence**

*Before you begin, you may want to watch the first two videos in the "Essence of linear algebra" series. You can find them both here (or search for the 1blue3brown channel on YouTube):*
https://www.youtube.com/playlist?list=PLZHQObOWTQDPD3MizzM2xVFitgF8hE_ab

**1.** Let $\bar{u} = \begin{pmatrix} -6 \\ -7 \\ 8 \end{pmatrix}$ and $\bar{v}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \bar{v}_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \bar{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$.

1.1. Write $\bar{u}$ as a linear combination of $\bar{v}_1, \bar{v}_2, \bar{v}_3$.

1.2. Can $\bar{v}_1$ be written as a linear combination of $\bar{v}_2, \bar{v}_3$? Justify your answer.

**2.** Show that the vectors $\bar{w}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \bar{w}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \bar{w}_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ form a spanning set of $\mathbb{R}^3$.

*Guidance*: Show that a general vector $\bar{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ can be written as a linear combination of $\bar{w}_1, \bar{w}_2\ \bar{w}_3$.

**3.** Let $\bar{v} = \begin{pmatrix} 2 \\ 7 \\ 8 \end{pmatrix}$ and $\bar{u}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \bar{u}_2 = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}, \bar{u}_3 = \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}$

3.1. Show that $\bar{v}$ **cannot** be written as a linear combination of $\bar{u}_1, \bar{u}_2, \bar{u}_3$.

3.2. Do $\bar{u}_1, \bar{u}_2, \bar{u}_3$ span $\mathbb{R}^3$?

**4.** Without any calculation, determine whether the following set of vectors in $\mathbb{R}^3$ is linearly dependent:
$\bar{u}_1 = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}, \bar{u}_2 = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \bar{u}_3 = \begin{pmatrix} 2 \\ 5 \\ 7 \end{pmatrix}, \bar{u}_4 = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$

**5.** The zero vector is a vector in which all entries are 0. It is usually denoted by 0 (and not $\bar{0}$). For example, the zero vector in 2D is $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.
Given the definition that we saw in class for linear dependence, prove that any set of vectors that includes the zero vector is linearly dependent.

**6.** Before the next class, please watch videos 3-4 in the "Essence of linear algebra" series:
* "Linear transformations and matrices"
* "Matrix multiplication as composition"
You can find them both here (or search for the 1blue3brown channel on YouTube):
https://www.youtube.com/playlist?list=PLZHQObOWTQDPD3MizzM2xVFitgF8hE_ab
The next class could start with a short quiz about these videos.

**Additional questions**

The following questions in the next page are not obligatory. Feel free to answer them if you feel that further practice will benefit you.

1. Let: $\bar{v} = \binom{1}{3}, \bar{u} = \binom{-1}{0}, \bar{w} = \binom{-2}{-6}$
   1.1. Draw the above vectors on the same coordinate system.
   1.2. Add a schematic illustration of $Sp(\bar{v})$
   1.3. Calculate $\bar{v} + 3\bar{u}$ and show that your answer agrees with the geometrical interpretation of vector summation.
2. Check your answer to question 2 in the main exercise for the specific vector $\bar{v} = \begin{pmatrix} 5 \\ -6 \\ 2 \end{pmatrix}$