Linear Algebra for Neuroscience (76992)

Exercise 07 – The inner product and orthogonality

1. Let $\bar{v} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}, \bar{w} = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$.

1.1. Calculate the projection of $\bar{v}$ onto $\bar{w}$.

1.2. Calculate the projection of $\bar{w}$ onto $\bar{v}$.

2. Let $\bar{u}, \bar{w} \in \mathbb{R}^2$ be two orthogonal vectors.
For this question only, assume that you don't know that orthogonal vectors satisfy $\bar{u}^T\bar{w} = 0$.

Use the Pythagorean theorem to show that $\bar{u}^T\bar{w} = 0$.

3. Let $\bar{x}$ be an $n \times 1$ vector and A be a $n \times n$ matrix.
What is the result of $\bar{x}^T A\bar{x}$? If it is a vector or matrix, write down the general term of the result. If it is a scalar, write down the result. You should use the Σ notation.

4. Let A be a matrix. Show that any vector $\bar{x} \in \ker(A)$ is orthogonal to any vector $\bar{a} \in rowsp(A)$ (remember, the row space of A is the space spanned by the rows of the matrix A).