Linear Algebra for Neuroscience (76992)

Exercise 10 – Complex vectors and matrices & Vector spaces

Complex vectors and matrices

1. As we saw in class, a general rotation-scaling matrix has the following form:

$$A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

1.1. Let λ be one of the complex eigenvalue of A.
Show that the scaling factor that A induces is $|\lambda|$ (the magnitude λ).

1.2. Assume that A rotates by $10°$ counter-clockwise, and scales by 1.5.
We define:

$$\bar{v}(t) = A\bar{v}(t-1)$$

Let $\bar{v}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
Draw a schematic representation of $\bar{v}(0), \bar{v}(1), \bar{v}(5), \bar{v}(9), \bar{v}(10), \bar{v}(36)$.

1.3. Repeat the previous question, but this time with $|\lambda| = 1$.

1.4. Repeat the previous question, but this time with $|\lambda| < 1$.

Vector spaces

2. True/false? Justify your answers.

2.1. Let A be a real $m \times n$ matrix. $rowsp(A)$ is a subspace of $\mathbb{R}^m$.

2.2. Let A be a real $m \times n$ matrix. $rowsp(A)$ is a subspace of $\mathbb{R}^n$.

2.3. The solution set for a nonhomogeneous system $A\bar{x} = \bar{b}$ is a subspace of $\mathbb{R}^n$.

2.4. The set of real 2×2 matrices with zero determinant ($\{A \in M_{2\times 2} | \det(A) = 0\}$) is a subspace of the space of real 2×2 matrices.

3. Let V be the vector space of real 2×2 matrices. Let U be the subspace of real symmetric 2×2 matrices.

3.1. Find $\dim(U)$.
To prove your answer, find a basis for U. Make sure to prove that it is indeed a basis (remember the two properties that every basis needs to fulfill).

4. Let $P_2(x)$ be the vector space of polynomials of degree smaller or equal to 2. The following polynomials form a basis for this space:

$$p_1 = x + 1$$
$$p_2 = x - 1$$
$$p_3 = x^2 - 2x + 1$$

Let $\bar{v} = 2x^2 - 5x + 9$ be a vector in this space. Find the coordinate vector (i.e., the coefficients) of $\bar{v}$ with respect to this new basis.