Linear Algebra for Neuroscience (76992)

Exercise 11 – The four fundamental subspaces & inner product spaces

1. Let P be a matrix such that $P^2 = P$.

1.1. Show that $(I - P)^2 = I - P$.

1.2. If P is a projection matrix onto $Im(A)$, then $I - P$ is a projection matrix onto _____.

2. In Ex04, Q3, you studied two systems of equations. For each system draw a schematic representation of the four fundamental subspaces:

- Clearly mark the dimensions of each subspace.
- Mark the vector $\bar{b}$ in the relevant position.

3. Let A be a 7x9 matrix with rank 5. What are the dimensions of the four subspaces?

4. Let A be a $m \times n$ matrix with rank r. Suppose there exists a vector $\bar{b}$ such that $A\bar{x} = \bar{b}$ has no solution.

4.1. What are the inequalities ($<, \leq,\ or =$) that must be true between m, n, and r?

4.2. How do you know that $A^T\bar{y} = 0$ has solutions other than $\bar{y} = 0$?

5. True or false?
If the matrices A and B have the same four fundamental subspaces, then A is a multiple of B.

<u>Inner product spaces</u>

6. Prove that the inner product over the complex numbers is linear in the second term.

7. Prove that for any inner product space, the norm satisfies:

$$||k\bar{v}|| = |k| \cdot ||\bar{v}||$$